

Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research

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Our overall intent is to clarify relations between the psychological constructivist, sociocultural, and emergent perspectives. We provide a grounding for the comparisons in the first part of the article by outlining an interpretive framework that we developed in the course of a classroom-based research project. At this level of classroom processes, the framework involves an emergent approach in which psychological constructivist analyses of individual activity are coordinated with interactionist analyses of classroom interactions and discourse. In the second part of the article, we describe an elaboration of the framework that locates classroom processes in school and societal contexts. The perspective taken at this level is broadly sociocultural and focuses on the influence of individuals' participation in culturally organized practices. In the third part of the article, we use the discussion of the framework as a backdrop against which to compare and contrast the three theoretical perspectives. We discuss how the emergent approach augments the psychological constructivist perspective by making it possible to locate analyses of individual students' constructive activities in social context. In addition, we consider the purposes for which the emergent and sociocultural perspectives might be particularly appropriate and observe that they together offer characterizations of individual students' activities, the classroom community, and broader communities of practice.

One of the most significant developments in American educational research during the past decade has been the increasingly prominent role played by both constructivist and sociocultural approaches. Initially, adherents to these two perspectives tended to argue for the hegemony of their own views. However, there appears to be a growing consensus that the perspectives are at least partially complementary (Cobb, 1994; Confrey, 1995; Hatano, 1993; Smith, 1995; Steffe, 1995). We contribute to this ongoing discussion in this article by exploring possible relations between sociocultural theory and various forms of constructivism. Our interest in these relations is pragmatically based and stems from our involvement in a classroom-based research and development project. In particular, we draw on several different theoretical viewpoints when addressing issues that arose while working with teachers and their students. It is in this sense that our views about possible relations between theoretical perspectives are rooted in our activity of attempting to understand what might

be going on in a range of specific teaching and learning situations. In the course of the discussion, we attempt to exemplify this grounding by outlining the settings in which the proposed relations first emerged for us.

In the first part of this article, we describe the interpretive framework that we currently use when analyzing teachers' and students' activity in the classroom. This framework represents an emergent, or social constructivist, approach that evolved from an initial psychological constructivist position. We outline the rationale for the framework by indicating the unanticipated problems that we found ourselves addressing and the interpretive stances that we eventually took. In the second part of this article, we describe how we subsequently found it necessary to extend this framework beyond the classroom level by drawing on sociocultural theory. In the third part of the article, we use the discussion of the framework as a backdrop against which to compare and contrast psychological constructivist, emergent, and sociocultural perspectives.

The approach we take in this article of attempting to ground theory in practice reflects the view that the relation between theory and practice is reflexive (Cobb, 1995; Lemke, 1995; Simon, 1995). Theory is seen to grow out of practice

and to feed back to inform and guide practice. This approach can be contrasted with more traditional styles of presentation in which the basic principles or tenets of theoretical positions are stated, and then implications are deduced for practice. As Schön (1983) observed, this rhetorical style elevates theory over practice and enacts a positivist epistemology of practice, thereby devaluing the relation between theory and practice as it is lived by reflective practitioners (Ball, 1993; Lampert, 1990; Simon & Blume, 1994). Furthermore, characterizations of this type tend to position researchers and practitioners in superior and subordinate roles as producers of theory and consumers of implications. In contrast, alternative styles of presentation that attempt to ground theory in practice suggest a more collaborative relationship between teachers and researchers in which their areas of expertise are seen as complementary rather than as hierarchically organized (Nicholls & Hazzard, 1993). In our view, accounts of this type have greater potential to contribute to current reform efforts.

THE INTERPRETIVE FRAMEWORK AT THE CLASSROOM LEVEL

The interpretive framework we outline was developed in the course of an ongoing program of *developmental research* in which instructional *development* and classroom-based *research* went hand in hand (cf. Gravemeijer, 1994). Developmental research as we define it here is therefore not synonymous with either child development research or research into the development of particular conceptions. The basic developmental research cycle is shown in Figure 1. Gravemeijer clarified that this cycle occurs at a variety of levels. In doing so, he took care to differentiate developmental research from the traditional formative-evaluation approach of implementing predetermined instructional activities and strategies and then evaluating their effectiveness. In the case of our own work, the most general goal was to investigate ways of proactively supporting elementary school students' mathematical development in the classroom. As part of this process, we and our colleagues developed both sequences of instruc-

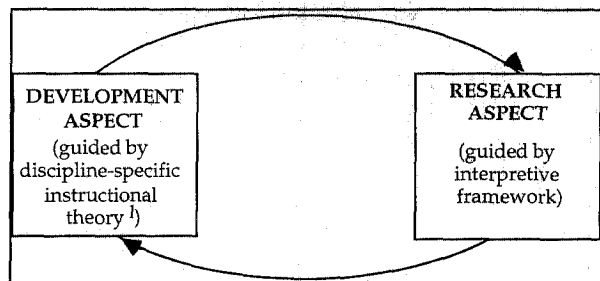


FIGURE 1 Phases of the developmental research cycle.

¹The domain-specific instructional theory referred to is that of realistic mathematics education developed at the Freudenthal Institute (Streefland, 1991; Treffers, 1987). This developmental research cycle is, in many ways, analogous to the mathematics teaching cycle described by Simon (1995).

tional activities for students and an approach to professional development for teachers. The general methodology employed was that of the classroom teaching experiment conducted in collaboration with a practicing teacher who was a member of the research and development team. In the past 9 years, we completed a series of these year-long experiments at the first-, second-, and third-grade levels.

At the outset, we intended to explain students' mathematical activity and learning in individualistic psychological terms. However, we soon concluded that such accounts were inadequate for the purposes of developmental research. As a consequence, one of our primary theoretical objectives became that of exploring ways to account for students' mathematical development *as it occurs in the social context of the classroom*. Analyses of this type are central to the second aspect of the developmental research cycle shown in Figure 1 and feed back to inform ongoing instructional development efforts. The interpretive framework we outline is a response to this issue of accounting for learning in social context. Although our focus was on students' learning, there is some indication that the framework can also be adapted to guide analyses of teachers' socially situated activity (McClain, 1995; Simon, 1995). For example, Simon analyzed his own teaching of mathematics to prospective teachers in order to develop a model of mathematics teaching that is informed by a social constructivist view of learning. McClain, for her part, identified aspects of effective reform teaching by analyzing the instructional practices of a first-grade teacher with whom we collaborated during a year-long teaching experiment.

These uses of the framework acknowledged, we want to avoid the essentialist implication that it might somehow capture the structure of individual and collective classroom activity independent of history, situation, and purpose. The most that is claimed is that we currently find the framework helpful when attempting to support reform in mathematics education. As a further caveat, we also clarify that when we speak of the framework at the classroom level, we do not mean this as a physical location. Instead, our intent is to indicate that explanations are formulated in terms of processes that are located at the classroom level—individual interpretations and actions, and face-to-face interactions and discourse. Explanations of this type do not make reference to students' participation in practices outside the classroom.

The interpretive framework at this level is shown in Figure 2. As the column headings *Social Perspective* and *Psychological Perspective* indicate, the framework involves the explicit coordination of two distinct theoretical viewpoints on classroom activity. The social perspective is an interactionist view of communal or collective classroom processes (Bauersfeld, Krummheuer, & Voigt, 1988). The psychological perspective is a psychological constructivist view of individual students' (or the teacher's) activity as they participate in and contribute to the development of these communal processes (von Glasersfeld, 1984, 1992). The coordination of interactionism and psychological constructivism is the primary de-

SOCIAL PERSPECTIVE	PSYCHOLOGICAL PERSPECTIVE
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

FIGURE 2 An interpretive framework for analyzing individual and collective activity at the classroom level.

fining characteristic of the version of social constructivism that we refer to as the *emergent approach* or the *emergent perspective* (Cobb & Bauersfeld, 1995).

As an initial orientation, consider the constructs listed under the Social Perspective heading: classroom social norms, sociomathematical norms, and classroom mathematical practices. These constructs denote three aspects of the classroom microculture that we found useful to distinguish. The column headed Psychological Perspective lists the psychological constructs that we took to be the individual correlates of these social constructs. Thus, each row of Figure 2 embodies a conjectured relation between an aspect of the classroom microculture and the activity of the individuals who participate and contribute to it. For example, it is apparent from the figure that we took individual students' beliefs about their own role, others' roles, and the general nature of mathematics in school to be the psychological correlates of general classroom social norms. This and the other two conjectured relations are predictive and are, therefore, open to empirical scrutiny. For example, the conjectured relation between classroom social norms and individual beliefs implies that a teacher who initiates and guides the renegotiation of classroom social norms is simultaneously supporting individual students' reorganization of the corresponding beliefs. It is this explanatory power that makes the framework particularly relevant to our purposes as we engage in classroom-based developmental research.

In the following paragraphs, we trace the history of the development of the various constructs. In doing so, we necessarily refer to specific classrooms in which we worked. The discussion of events in those classrooms is relatively brief because our goal is to develop the rationale for the framework rather than to present detailed analyses of either these events or of the collaborating teachers' complex and highly demanding instructional practices.

Classroom Social Norms

When we conducted our first classroom teaching experiment during the 1986–87 school year, we initially viewed learning in almost exclusively psychological constructivist terms. The classroom teaching methodology we used was originally devised as an extension of the constructivist teaching experiment in which the researcher interacts one-on-one with a single child and attempts to influence the child's constructive activities (Cobb & Steffe, 1983; Steffe, 1983). In the case of the constructivist teaching experiment, the goal was to account for the child's development of increasingly powerful mathematical ways of knowing by analyzing the cognitive restructurings he or she made while interacting with the researcher. In a similar manner, we intended to account for individual children's learning in the classroom by analyzing the conceptual reorganizations they made while interacting with the teacher and their peers. With hindsight, it is apparent that the relation between social interaction and children's mathematical development implicit in this approach was neo-Piagetian. We assumed that conflicts in individual students' mathematical interpretations might give rise to internal cognitive conflicts, and we assumed that these would precipitate mathematical learning (cf. Doise & Mugny, 1984; Perret-Clermont, 1980). In this account, social interaction was viewed as a catalyst for otherwise autonomous psychological development because it influenced the process of mathematical development but not its products, increasingly sophisticated mathematical ways of knowing.

The first unanticipated issue that we addressed in this initial classroom teaching experiment arose within the first few days of the school year. The second-grade teacher with whom we worked engaged her students in both collaborative small-group work and whole-class discussions of their mathematical interpretations and solutions. However, it soon be-

came apparent that the teacher's expectation that the children would publicly explain how they had actually interpreted and solved tasks ran counter to their prior experiences of class discussions in school. The students had been in traditional classrooms during their first-grade year and seemed to take it for granted that they were to infer the responses the teacher had in mind rather than to articulate their own understandings. The teacher coped with this conflict between her own and the students' expectations by initiating a process that we subsequently came to term *the renegotiation of classroom social norms*. Examples of social norms for whole-class discussions that the teacher framed as explicit topics for negotiation included explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations in which a conflict in interpretations or solutions had become apparent. In general, social norms can be seen to delineate the classroom participation structure (Erickson, 1986; Lampert, 1990).

A detailed account of the renegotiation process in this second-grade classroom was given elsewhere (Cobb, Yackel, & Wood, 1989). For our purposes, it suffices to note that social norms are not psychological processes or entities that can be attributed to any particular individual. Instead, they characterize regularities in communal or collective classroom activity and are considered to be jointly established by the teacher and students as members of the classroom community. We would, therefore, question accounts framed in individualistic terms in which the teacher is said to establish or specify social norms for students. To be sure, the teacher is necessarily an institutionalized authority in the classroom (Bishop, 1985). He or she is seen to express that authority in action by initiating, guiding, and organizing the renegotiation process. However, the students are also seen to play their part in contributing to the evolution of social norms.² One of our primary conjectures is, in fact, that in making these contributions, students reorganize their individual beliefs about their own role, others' roles, and the general nature of mathematical activity (Cobb et al., 1989). As a consequence, we take these beliefs to be the psychological correlates of the classroom social norms.

It is important to clarify that, in the view we are advancing, neither the social norms nor individual students' beliefs are given primacy over the other. Thus, for example, it is neither a case of a change in social norms causing a change in students' beliefs nor a case of students first reorganizing their beliefs and then contributing to the evolution of social norms. Instead, social norms and beliefs are seen to be reflexively related such that neither exists independently of the other. We can further clarify our position by noting with Whitson (in press) that human activities in the classroom can be described

and analyzed in a variety of different terms depending on the issues at hand. The psychological and sociological perspectives are two ways of describing that we found particularly relevant for our purposes. In conducting a social analysis from the interactionist perspective, we document the evolution of social norms by taking an analytical position as observers who are located outside the classroom community. In contrast, when we conduct a psychological constructivist analysis, we focus on individual students' activity as they participate in communal processes and document their reorganization of their beliefs. From one perspective, we describe the joint or collective processes constituted by actively cognizing individuals and, from the other, we describe the interpretations and construals of individuals as they participate in those collective processes. Metaphorically speaking, communal processes that are in the foreground when we adopt a social perspective become part of the unarticulated background against which we conduct psychological analyses, and vice versa. The social constructivist, or emergent, approach to which we subscribe attempts to coordinate these two ways of analyzing classroom activity and treats them as complementary. In this joint perspective, classroom social norms are seen to evolve as students reorganize their beliefs, and, conversely, the reorganization of these beliefs is seen to be enabled and constrained by evolving social norms.

Sociomathematical Norms

Thus far, in describing our initial interest in classroom social norms, we explained why we found it necessary to go beyond an exclusively individualistic psychological perspective. We again stress that we did not analyze these norms as an end in itself. Instead, our motivation was to account for students' mathematical development as it occurred in the social context of the classroom. In this regard, one aspect of our analysis of social norms that proved disquieting, given our agenda as mathematics educators, was that it was not specific to mathematics, but applied to almost any subject matter area. For example, one would hope that students might challenge each other's thinking and justify their own interpretations in science and literature lessons as well as in mathematics lessons. We attempted to address this limitation of our work by shifting our focus in subsequent analyses to the normative aspects of whole-class discussions that are specific to students' mathematical activity (Lampert, 1990; Voigt, 1995; Yackel & Cobb, 1996). Examples of such sociomathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation.

As part of the process of guiding the development of an inquiry approach to mathematics in their classrooms, the teachers with whom we worked regularly asked the students if anyone had solved a task a different way and then either sanctioned or implicitly delegitimized contributions that they did not consider to be mathematically different from those

²Cooney's (1985) analysis of Fred, a beginning mathematics teacher, provides an excellent illustration of this point. In our terms, Cooney documented the difficulties that Fred encountered when he attempted to initiate the renegotiation of social norms and institute a problem-solving approach.

that had been given by other students. It was while analyzing classroom interactions of this type that sociomathematical norms first emerged as an explicit focus of interest for us. The analysis indicated that, on the one hand, the students did not know what would count as a mathematically significant difference in their explanations until the teacher and other students judged that some of their contributions, but not others, were different. Consequently, in responding to the teacher's requests for a different solution, the students were both learning what counted as a mathematically significant difference and helping to interactively constitute what counted as a mathematically significant difference in their classroom. On the other hand, the teachers in these classrooms were themselves attempting to develop an inquiry form of practice and had not, in their prior years of teaching, asked students to explain their thinking. Consequently, the experiential basis from which they attempted to anticipate students' contributions was extremely limited. Furthermore, they had not necessarily decided in advance what would constitute a mathematically significant difference. Instead, the teachers seemed to clarify their own understanding of what should count as a mathematical difference as they interacted with their students. Viewed in this way, the sociomathematical norm of mathematical difference appeared to emerge in the course of joint activity via a process of implicit negotiation. A similar conclusion also holds for the other sociomathematical norms we analyzed (Yackel & Cobb, 1996).

The analysis of sociomathematical norms proved to be pragmatically significant because it helped us understand the process by which the teachers with whom we collaborated fostered the development of intellectual autonomy in their classrooms. This issue is particularly significant to us because the development of student autonomy was an explicitly stated goal of our work in classrooms from the outset. However, we originally characterized intellectual autonomy in individualistic terms and spoke of students' awareness of and willingness to draw on their own intellectual capabilities when making mathematical decisions and judgments. We contrasted this view of intellectual autonomy with intellectual heteronomy, wherein students rely on the pronouncements of an authority to know how to act appropriately (Kamii, 1985; Piaget, 1973). As part of the process of supporting the growth of autonomy, the teachers with whom we worked guided the development of a community of validators in their classrooms such that claims were established by means of mathematical argumentation rather than by appealing to an authority such as that teacher or a textbook. However, for this to occur, it was not sufficient for the students to merely learn that they should make a wide range of mathematical contributions. Mathematically unproductive discussions occurred unless they also had developed personal ways of judging that enabled them to know-in-action both when it was appropriate to make a mathematical contribution and what constituted an acceptable contribution. This required, among other things, that the students could themselves judge what counted as a different

mathematical solution, an insightful mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation. However, these were precisely the types of judgments that are negotiated when establishing sociomathematical norms. Therefore, we conjecture that students construct specifically mathematical beliefs and values that enable them to act as increasingly autonomous members of classroom mathematical communities as they participate in the renegotiation of sociomathematical norms (Yackel & Cobb, 1993). These beliefs and values, it should be noted, are psychological constructs and constitute what the National Council of Teachers of Mathematics (1991) called a *mathematical disposition*. We take them to be the psychological correlates of sociomathematical norms and consider the two to be reflexively related (see Figure 2).

It is apparent from the account we have given that we revised our conception of intellectual autonomy in the course of the analysis. At the outset, we defined autonomy in psychological terms as a characteristic of individual activity. However, by the time we completed the analysis, we came to view autonomy as a characteristic of an individual's participation in a community. Thus, although the development of autonomy continues to be a central pragmatic goal for us, we redefined our view of what it means to be autonomous by going beyond our original psychological constructivist position. This shift in perspective enabled us to be more effective in helping teachers support the development of autonomy in their classrooms (McClain, 1995).

Classroom Mathematical Practices

The third aspect of the interpretive framework, that concerning classroom mathematical practices, was motivated by the realization that one can talk of the mathematical development of a classroom community as well as of individual children. For example, in the second-grade classrooms in which we worked, various solution methods that involve counting by 1s were established mathematical practices at the beginning of the school year. Some of the students were also able to develop solutions that involved the conceptual creation of units of 10 and 1. However, when they did so, they were obliged to explain and justify their interpretations of number words and numerals. Later in the school year, solutions based on such interpretations were taken as self-evident by the classroom community. The activity of interpreting number words and numerals in this way became an established mathematical practice that no longer needed justification. From the students' point of view, numbers simply were composed of 10s and 1s—it was a mathematical truth.

This illustration from the second-grade classrooms describes a global shift in classroom mathematical practices that occurred over a period of several weeks. An example of a detailed analysis of evolving classroom practices can be found in Cobb, Gravemeijer, Yackel, McClain, and Whitenack (in press). We contend that analyses of this type are appropriate

for the purposes of developmental research because they document instructional sequences as they are realized in interaction in the classroom. They, therefore, draw together the two general aspects of developmental research, instructional development and classroom-based research, and feed back to inform ongoing development efforts (see Figure 1).

Analyses of this type are also of theoretical significance because they bear directly on the issue of accounting for mathematical learning as it occurs in the social context of the classroom. Viewed against the background of classroom social and sociomathematical norms, the mathematical practices established by the classroom community can be seen to constitute the immediate, local situations of the students' development. Consequently, in identifying sequences of such practices, the analysis documents the evolving social situations in which students participate and learn. Individual students' mathematical conceptions and activities are taken as the psychological correlates of these practices, and the relation between them is considered to be reflexive. In particular, we consider that students actively contribute to the evolution of classroom mathematical practices as they reorganize their individual mathematical activities and, conversely, that these reorganizations are enabled and constrained by the students' participation in the mathematical practices.

As a point of clarification, we stress that psychological analyses typically reveal qualitative differences in individual children's mathematical interpretations even as they participate in the same mathematical practices. In general, analyses conducted from the psychological constructivist perspective bring out the heterogeneity in the activities of members of a classroom community. In contrast, social analyses of classroom mathematical practices conducted from the interactionist perspective bring out what is jointly established as the teacher and students coordinate their individual activities. In drawing on these two analytic perspectives, the emergent approach takes both the individual and the community as points of reference. This approach seeks to analyze both the development of individual minds and the evolution of the local social worlds in which those minds participate (cf. Balacheff, 1990).

Summary

We pause to make two points about the interpretive framework outlined thus far. The first concerns a possible misinterpretation. In the past, we sometimes have been interpreted as saying that students' mathematical activity is essentially psychological and individualistic, but that it is constrained by social and cultural processes, such as social norms. We, therefore, stress that we consider students' mathematical activity to be social through and through because it does not develop apart from their participation in communities of practice. More generally, our intent is not to classify teachers' and students' activities into those that are intrinsically individual and those that are intrinsically communal. Instead, our proposal is to *coordinate* analyses of classroom processes that are conducted in psychological and social terms.

The second point is methodological and concerns the products of developmental research. These products, it should be noted, include sequences of instructional activities as well as analyses of students' learning in social context as the sequences are realized in interaction in the classroom. A central assumption of developmental research is that productive patterns of learning can occur when an instructional sequence is enacted in other classrooms. However, as we know only too well, the history of educational research in general, and mathematics education in particular, is replete with more than its share of disparate and irreconcilable findings. From the emergent perspective, a primary source of difficulty is that the independent variables of traditional experimental research are relatively superficial and have little to do with either context or meaning. Such approaches are difficult to justify if one follows Lemke (in press) and considers that the ecology of the classroom is semiotic and involves meaning-making in which one thing is taken as a sign for another. Lemke called systems with semiotic ecologies *ecosocial systems*. From this point of view, students are seen to always perceive, act, and learn by participating in the self-organization of systems that are larger than themselves, such as the communities of practice established in classrooms. Learning can therefore be characterized as "an aspect of self organization, not just of the human organism as a biological system, but of ecosocial systems in which the organism functions as a human being" (Lemke, in press). It is precisely this sense of participation in an evolving community of practice that is typically ignored in traditional experimental research.

In our view, the primary concern is not so much that past findings have been disparate, but that they have been irreconcilable—it is not possible to account for differences in findings when different groups of students received supposedly the same instructional treatment. In contrast to traditional experimental research, the challenge as we see it is not that of replicating instructional sequences by ensuring that they are enacted in the same way in different classrooms. The conception of teachers as professionals who continually adjust their plans on the basis of ongoing assessments of individual and collective activity in the classroom in fact suggests that complete replicability is neither desirable nor, perhaps, possible (cf. Ball, 1993; Simon, 1995.) The challenge, as it is construed from the emergent perspective, is instead to develop ways of analyzing both instructional sequences and students' participation in them as they are realized in interaction in different classrooms. In this regard, we note that the framework as outlined thus far illustrates one possible way to organize analyses of both the classroom ecosocial system and the activity of the students (and teacher) who contribute to its development. For example, we suggested that the constructs of social norms, sociomathematical norms, and classroom mathematical practices address aspects of the classroom microculture that are relevant to the purposes of developmental research. An analysis of classroom events organized in this way might, therefore, relate the emerging patterns of students'

learning to their participation in sequences of instructional activities as they are realized in interaction. In addition, the teacher's role in guiding and organizing the development of both the classroom ecosocial system and the activity of the children who participate in it could become an explicit object of analysis, as could the broader institutional contexts in which such systems are embedded.

We note that the intent of these comments is not to recommend that others should necessarily use the specific framework we outlined. Instead, our intent is to illustrate the potential contribution of a framework of this type that is concerned with context and meaning. In particular, such a framework might support greater precision in developmental research by making it possible to compare, contrast, and relate different enactments of instructional sequences. This in turn would facilitate disciplined, systematic inquiry that embraces the messiness and complexity of the classroom.

THE INTERPRETIVE FRAMEWORK AT THE SCHOOL AND SOCIETAL LEVELS

In the course of our ongoing research and development activities, we often were able to develop explanations that proved adequate for our purposes by referring solely to classroom processes. There were, however, occasions when we found it essential to take account of the broader institutional contexts in which such systems are embedded. The elaborated

version of the interpretive framework shown in Figure 3 synthesizes our reflections on these experiences. The two inner boxes headed Psychological Perspective and Interactionist Perspective together represent a reconfiguration of the framework at the classroom level as it was shown in Figure 2. The intent of Figure 2 was to highlight conjectured relations between the classroom microculture and the activities of individual students who participate in it. In contrast, the arrangement of the inner boxes in Figure 3 emphasizes that, from the interactionist perspective, students' individual activities are framed in terms of their participation in the practices of the classroom community. Conversely, participation in these practices constitutes the background against which individual students' activity is analyzed from the psychological perspective. As we noted, the emergent approach explicitly coordinates these two perspectives.

In Figure 3, the outer two boxes correspond to norms and practices at the school and societal levels, respectively. It was when we took account of these processes that we found ourselves adopting a sociocultural perspective. In interpreting Figure 3, it is important to note that the apparent nesting of perspectives does *not* imply that the sociocultural perspective subsumes the interactionist and psychological perspectives. Instead, our intent is to indicate that the three perspectives frame individual activity differently. Thus, from the sociocultural perspective, individual students' activities are located in broader institutional settings. In contrast, an interactionist perspective frames their activity in terms of participation in

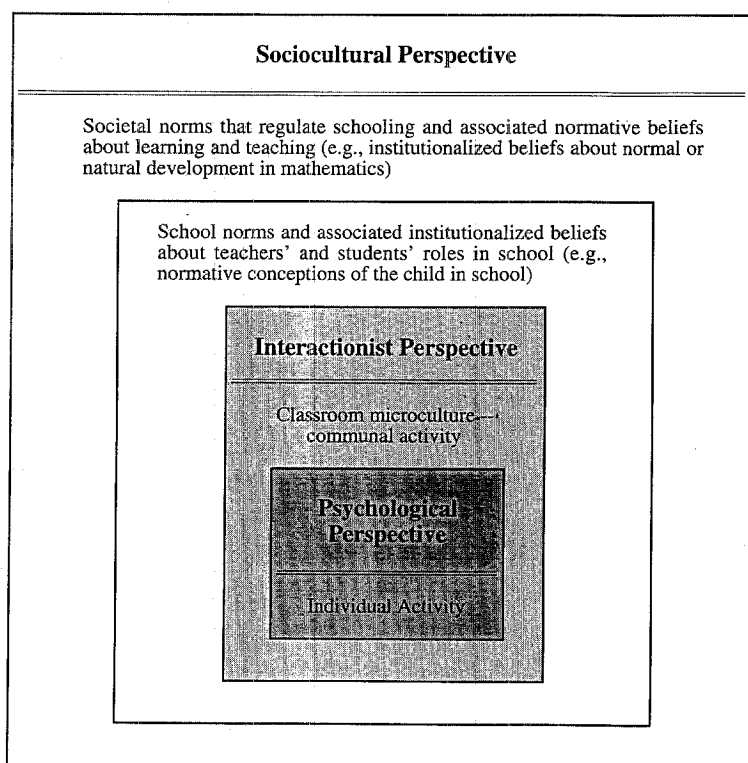


FIGURE 3 An elaboration of the interpretive framework.

the practices of the local community, whereas a psychological perspective focuses on individual interpretations and activity per se. We return to this issue of different framings in the final section of this article when we further clarify distinctions between the perspectives. First, however, we provide a grounding for the elaborated framework.

School Level

The need to take account of broader institutional contexts first became apparent to us when we attempted to account for our experiences of working with approximately 50 first-, second-, and third-grade teachers at two action research sites. One of these sites was rural-suburban, whereas the other served an almost exclusively inner-city student population.³ Our overall goal was to help these teachers revise the ways in which they taught mathematics. To this end, we formulated an initial approach to teacher development at the rural-suburban site, where it proved to be reasonably successful. Our first priority when working with the teachers at this site was to help them make aspects of their textbook-based instruction problematic. We reasoned that only then would they have reason and motivation to want to reform their instructional practices while working with us. To this end, we used video recordings of both individual interviews and classroom episodes to explore the consequences of traditional instruction. We previously documented the success of this approach at the rural-suburban site. We observed, for example, that the teachers

began to differentiate between correct adherence to accepted procedures and [children's] mathematical activity that expressed conceptual understanding.

As the teachers began to question the adequacy of textbook instructional activities and their current ways of teaching, they were then willing to consider alternative instructional activities designed to encourage meaningful mathematical activity. In doing so, they demonstrated the value they placed on children's mathematical sense-making. We did not have to convince them that children should learn with understanding. Rather, they had assumed that this kind of learning was occurring in their classrooms. *A shared desire to facilitate meaningful learning and a general concern for children's intellectual and social welfare constituted the foundation upon which we and the teachers began to mutually construct a consensual domain* [italics added]. (Cobb, Wood, & Yackel, 1990, p. 140)

With our support during the school year, the 20 teachers referred to in the preceding passage radically revised the way they taught mathematics.

³In using the designations *rural-suburban* and *inner-city*, we do not mean to imply that the schools in which we worked were typical or representative of other schools in similar locations. Furthermore, the very notion that any site is representative of schools in such locations is open to dispute.

Shortly after this passage was written, we began working at the inner-city site. It soon became apparent that our initial approach to teacher development was not viable at that site. For the most part, an exploration of the consequences of traditional instruction did not lead these teachers to question their current instructional approaches. Therefore, it seemed to us at the time that whereas the teachers at the rural-suburban site assumed without question that students should learn mathematics with understanding, the beliefs and values of the teachers at the inner-city site did not appear to be in conflict with traditional instructional practices. Our subsequent efforts to support these inner-city teachers were more successful, and several of them did, in fact, go on to develop forms of practice that were compatible with current reform recommendations in mathematics education. However, as we documented elsewhere, differences were still evident because the processes by which these teachers reorganized their practices differed significantly from those of the teachers at the rural-suburban site (Feikes, 1992; Yackel & Cobb, 1993).

In reflecting on these experiences, we subsequently realized that assumptions we initially considered to be self-evident in fact reflect our culturally specific beliefs and values. After working with teachers at the rural-suburban site, we wrote that "a shared desire to facilitate meaningful learning and a general concern for children's intellectual and social welfare" (Cobb, Wood, & Yackel, 1990, p. 140) constituted the foundation on which we and the teachers developed a basis for communication. At the time we wrote this statement, we assumed unquestioningly that engaging children in what for us counts as meaningful learning would necessarily be viewed as contributing to their welfare. However, our experiences at the inner-city site led us to reconsider this assumption.

Observations made at the inner-city site during both classroom mathematics lessons and teacher-induction sessions indicated that these teachers were deeply concerned about their students' intellectual and social welfare. However, there appeared to be crucial differences in what counted as intellectual and social welfare at the two sites (Yackel & Cobb, 1993). In particular, a disciplined environment seemed to be highly valued by teachers and administrators at the inner-city site. In addition, we did not observe instances in which the reasons for school rules were discussed with students. Thus, although there were discussions of whether a rule had been violated in particular instances, neither the appropriateness of the rules nor reasons for complying with them seemed to be topics of conversation.

In accounting for these differences between the two sites, we came to the view that *what it means to be a child in school* is constituted by pedagogical communities (Banks, 1995; Walkerdine, 1988). This notion does not, therefore, appear to be fixed and universal but is instead continually regenerated by the members of a pedagogical community as they participate in the practices of schooling. At the inner-city site in which we worked, for example, to be a child in school was to follow specific rules and instructions. Furthermore, to under-

stand was to be able to verbalize relevant rules. Consequently, adults showed their concern for children's welfare by helping them learn to follow and verbalize rules. It could well be the case that in guiding the development of a disciplined school environment, these teachers were attempting to provide students with a safe and secure setting for learning. The crucial point for our purposes is that there was no conflict at this site between the consequences of traditional mathematics instruction and the institutionalized views about what it meant to be a child in school. This in turn implied that the teachers had no reason to revise their current instructional practices.⁴

It is apparent that in the course of this discussion we found it useful for our purpose to view the teachers as representatives of particular communities of practice. This, of course, is not to deny that there were significant differences in the beliefs and instructional practices of individual teachers at each site. As we will see, this approach of characterizing individuals in terms of their participation in communities of practice is characteristic of a sociocultural perspective. With regard to the implications of the analysis, we observed that core beliefs and values implicit in current reform recommendations were compatible with those of the teachers at the rural-suburban site but conflicted with those of teachers at the inner-city site. This observation raises the possibility that reform efforts in which mathematics educators assume that their culturally situated commitments are universal might well result in even greater disparities in the types of mathematics education that children experience than is currently the case. We, therefore, follow Apple (1992) in calling for mathematics educators to explicate the ideological assumptions underpinning their reform recommendations. Only then might we be able to guard against the possibility that we will unknowingly foster even greater inequities.

Societal Level

The grounding for the discussion of practices at this level is provided by an analysis reported by Yang and Cobb (1995). At the outset, our goal was simply to build on previous investigations of the mathematics achievement of Asian and American students by comparing the arithmetical learning of children in Taiwan and the United States. However, in the course of the analysis, we came to the view that children in the two countries were participating in very different types of learning activities, and that these activities were culturally organized at the societal level.

With regard to the specifics of the investigation, the analy-

sis covered preschool through second grade and dealt with arithmetical developments up to and including the construction of place-value conceptions. Consistent with previous investigations, an analysis of video recorded, individual interviews indicated that there were significant differences in the quality of the two groups' arithmetical conceptions that favored the Taiwanese children (cf. Stevenson & Lee, 1990). In addition, an interactional analysis of classroom video recordings made in the two countries indicated that there were important differences in the obligations that the children had to fulfill to appear competent (cf. Stigler, Fernandez, & Yoshida, 1992). However, the most relevant differences for our purposes were those between the sequences of learning activities in which the children in the two countries participated. These sequences were identified by analyzing textbooks and by interviewing parents and teachers of the kindergarten, first-, and second-grade students. The issues addressed in these interviews included the types of learning activities that the teachers and parents considered most important for children's arithmetical development, the specific concepts and methods that children were expected to develop, the extent to which children needed either assistance or directed instruction, and the parents' and teachers' expectations for children's competencies at various age and grade levels.

The analysis indicated that there were important differences in the teachers' and parents' expectations for both the learning trajectories that the children would follow and the competencies they would develop, and in the extent to which the adults believed that it was necessary to provide direct support. In addition, there appeared to be differences in the internal consistency and coherence of the sequences of learning activities in the two countries. The American learning activities appeared to involve a major discontinuity in that the children's initial experiences in situations involving single-digit numbers did not appear to constitute a basis for their subsequent construction of place-value conceptions. Significantly, the American teachers and parents considered that place value was a challenging concept and that it should be delayed until the second grade. In addition, the American teachers unequivocally stated that direct instruction was required. By way of contrast, the culturally organized learning activities in Taiwan did not appear to have such contradictions. Further, the Taiwanese parents and teachers treated place-value conceptions as relatively unproblematic developments that should begin in kindergarten. The tasks they posed and the questions they asked both seemed to reflect the view that it is natural for children to conceptualize numbers as composed of 10s and 1s at a relatively early point in their arithmetical development. In addition, they did not consider this phase in children's arithmetical development to require direct instruction.

It is apparent from the analysis that the culturally organized learning activities in which the Taiwanese students participated tended to enable the development of conceptual understanding in arithmetic to a far greater extent than did the

⁴We were asked on several occasions whether the differences between the school communities reflect differences in the wider communities in which they were embedded. It would be inappropriate for us to address this issue for ethical reasons that pertain to the nature of the relationships we established with teachers and administrators at the two sites. As a consequence, a level corresponding to the wider community beyond the school is not included in Figure 3.

learning activities in which the American students participated. Furthermore, these differences in learning activities appeared to both corroborate and be supported by differences in the American and Taiwanese parents' and teachers' beliefs about what constitutes normal or natural development when children learn arithmetic. For example, the American parents and teachers had good reasons for believing that place value was a relatively late development. This belief in turn sustained pedagogical practices in which place value was experienced as a relatively challenging concept. Similarly, the Taiwanese teachers' and parents' beliefs were both expressed in and corroborated by the culturally organized learning activities in which they and their children participated. It, therefore, seems reasonable to characterize these two contrasting sets of beliefs about normal development as culturally situated social constructions that were reflexively verified in practice.

Many of the differences that we identified between mathematics education in the United States and Taiwan were reported elsewhere in the literature. If our analysis makes a contribution, it is to point to institutionalized beliefs and practices at the societal level that are specific to the development of particular mathematical conceptions. In the case of arithmetic, the analysis suggests that American reformers are challenging *regimes of truth* (Walkerdine, 1988) that define what counts as the natural state of affairs when they recommend significant changes in the sequencing and organization of the arithmetic curriculum. We speculate that similar conclusions hold for other areas of mathematics, including algebra, in which relatively radical curriculum reforms have been proposed. In each of these cases, the reform process entails a remaking of what is taken as normal or natural in students' mathematical development. From this point of view, the challenges of reform do indeed seem daunting. However, we contend that the integrity of reform efforts is threatened if we focus it narrowly on curriculum reform and fail to locate it in a broader cultural context by considering the regimes of truth that sustain current practices.

The general approach that we took when conducting the analysis was consistent with a sociocultural perspective because the American and Taiwanese children's contrasting arithmetical competencies were accounted for in terms of their participation in different sequences of culturally organized learning activities (cf. Cole, 1990; Lave & Wenger, 1991; Rogoff, 1994). Explanations of this type can be contrasted with an alternative orientation consistent with mainstream American psychology in which culture is treated as a cluster of variables that influences the course of essentially individualistic psychological processes. It should also be noted that the characterization of beliefs about psychological development as social constructions applies as much to widely accepted academic theories as it does to so-called folk theories (Lave, 1988; Newman, Griffin, & Cole, 1989). This, of course, does not imply that academic theories are mere myths or fictions, or that they are nothing more than arbitrary social conventions. Instead, our point is that these theories are

culturally situated and their development is guided by particular concerns and interests (Barnes, 1977). In our own case, for example, we came to see the emergent approach we outlined as grounded locally in the practices of developmental research and as located more globally in an encompassing activity system that constitutes schooling in the United States.

Summary

The elaborated version of the interpretive framework shown in Figure 3 emerged relative to our purposes and offers a way to organize analyses conducted from different theoretical perspectives. In describing the framework, we attempt to clarify how we found ourselves adopting several of the different perspectives in the course of our work with teachers and their students. The sociocultural perspective came to the fore when we considered practices at the school and societal levels, whereas the psychological constructivist and interactionist perspectives that together constitute the emergent approach were prominent when we focused on classroom processes. Significantly, we did not consider ourselves to be intellectually schizophrenic and did not experience any conflicts or theoretical anomalies when we conducted analyses from these various perspectives. In the final section of this article, we step back to compare and contrast the perspectives more directly, thereby articulating theoretical coordinations that were initially implicit in our activity of conducting classroom-based research.

COORDINATING PERSPECTIVES

Psychological Constructivism and the Emergent Perspective

We saw that the emergent perspective coordinates psychological constructivism with interactionism. This coordination can be clarified by considering, as an illustration, a situation in which a researcher is interacting with one student, perhaps conducting an interview or a one-on-one teaching session. To the extent that a psychological constructivist analysis takes account of the interaction, the primary focus is on the student's interpretations of the researcher's actions. An analysis of this type is made from inside the interaction and is concerned with the ways in which the student modifies his or her activity while interacting with the researcher. In contrast, an interactional analysis is made from the outside and makes the interaction between the student and the researcher an object of analysis. The focus is on patterns and regularities in their interactions and on the consensual meanings that emerge between them rather than on the student's personal interpretations. As Voigt (1994) made clear, these consensual meanings are not psychological elements that capture the partial match of individual interpretations but are, instead, located at the level of interaction. We exemplified this notion during the discussion of classroom mathematical practices when we described how nu-

merical interpretations involving 10s and 1s became institutionalized in a second-grade classroom. In doing so, we differentiated between taken-as-shared consensual meanings and individual students' various personal meanings.

Despite claims made to the contrary, we contend that researchers who typically take an individualistic focus are not conducting an interactional analysis merely because the students whose activity they are analyzing happen to be interacting with others. The researcher is conducting a psychological analysis as long as he or she focuses on the activity of each of the interacting individuals and fails to take their joint or communal activity as an explicit object of analysis (Blumer, 1969). By the same token, it is clear that the emergent approach does not merely involve bolting a social component onto an otherwise unchanged psychological approach (cf. Ernest, 1994). Instead, the relation between the interactionist and psychological constructivist perspectives is considered to be reflexive. The characterization of learning as an individual constructive activity is, therefore, relativized because these constructions are seen to occur as students participate in and contribute to the practices of the local community.

The comments made thus far do not delegitimize psychological analyses of, say, interviews or one-on-one teaching sessions. However, we do question the assumption that such analyses can, in principle, capture individual students' conceptual understandings independently of situation and purpose. From the emergent perspective, interviews are social events in which the researcher and student negotiate their roles, their interpretations of tasks, and their understanding of what counts as a legitimate solution and an adequate explanation (Mishler, 1986; Schoenfeld, 1987; Voigt, 1995). As a consequence, we argue that it is important to view the students' activity as being socially situated even in settings such as interviews, which are typically associated with psychological paradigms. The psychological analysis would then be conducted against the background of a social analysis that documents the interactively constituted situation in which the student is acting.

We argued that the emergent approach is consistent with the purposes of classroom-based developmental research. We also clarified that analyses conducted in line with this approach can give greater prominence to either the psychological or the interactionist perspective, depending on the issues and purposes at hand. In each case, one perspective comes to the fore against the background of the other. This reciprocity between the psychological and social in turn serves to differentiate the emergent approach from sociocultural approaches.

Emergent and Sociocultural Perspectives

The emergent and sociocultural perspectives have a number of points in common. For example, both reflect the view that learning and understanding are inherently social and cultural

activities. The two positions, therefore, reject the view that social interactions serve as a catalyst for otherwise autonomous intellectual development. In addition, both attend to the role of symbols and artifacts in conceptual development. A primary difference between the two perspectives concerns the way in which activity is conceptualized or framed. Analyses conducted from the emergent perspective typically take the practices of the local community, such as that established in a classroom, as a point of reference. In contrast, analyses conducted from the sociocultural perspective typically view individuals as participating in broader sociocultural practices. In the following paragraphs, we identify additional differences between the two perspectives by considering learning, teaching, and semiotic mediation. We then consider situations in which one perspective or the other might be more appropriate for particular purposes.

We saw that from an emergent perspective, learning is a constructive process that occurs while participating in and contributing to the practices of the local community. In the case of the interpretive framework, for example, students were seen to actively construct their mathematical ways of knowing as they participated in the mathematical practices of the classroom community. The link between collective and individual processes in this approach is, therefore, indirect because participation enables and constrains learning but does not determine it. Participation is, therefore, seen to constitute the conditions for the possibility of learning (Krummheuer, 1992). In contrast, a Vygotskian perspective such as that advanced by van Oers (1996) treats the link between collective processes and individual processes as a direct one: The qualities of students' thinking are *generated by* or *derived from* the organizational features of the social activities in which they participate. This conjectured direct linkage allows sociocultural theorists to be more directive when making instructional recommendations. For example, van Oers suggested that students should imitate culturally established mathematical practices when they interact with the teacher or more capable peers. He went on to argue that help should be gradually withdrawn so that students take over functions they initially could not perform alone, thereby internalizing the cultural activity. This recommendation instantiates Vygotsky's (1960) frequently cited general genetic law of cultural development:

Any higher mental function was external and social before it was internal. It was once a social relationship between two people. ... We can formulate the general genetic law of cultural development in the following way. Any function appears twice or on two planes. ... It appears first between people as an intermental category, and then within the child as an intramental category. (pp. 197-198)

The contrasting emphases of the sociocultural and emergent perspectives are reflected in differing characterizations of the teacher's role in proactively supporting students' mathematical development. In sociocultural accounts, the

teacher is typically portrayed as a representative of society who supports students' reconstruction of culturally approved meanings (cf. Forman, 1996). This view leads to a treatment of negotiation that is partially at odds with emergent accounts of communication. From the emergent perspective, negotiation is a process of mutual *adaptation* that gives rise to shifts and slides of meaning as the teacher and students coordinate their individual activities, in the process constituting the practices of the classroom community. However, from the sociocultural perspective, negotiation is a process of mutual *appropriation* in which the teacher and students continually co-opt or use each others' contributions (Newman, Griffin, & Cole, 1989). The teacher is, therefore, typically expected to insert culturally approved insights that students can co-opt, and to appropriate students' actions into the wider system of mathematical practices that they are to reconstruct. In this account, the teacher negotiates with students in order to mediate between their personal meanings and established cultural meanings. However, in the emergent approach, it is the local classroom community, rather than the mathematical practices institutionalized by wider society, that is taken as the immediate point of reference. From this point of view, the teacher's role while negotiating with students is characterized as that of proactively supporting both students' individual constructions and the evolution of classroom mathematical practices so that students increasingly become able to participate effectively in the mathematical practices of the wider society. In general, whereas sociocultural approaches frame instructional issues in terms of the *transmission* of culture from one generation to the next, the emergent perspective frames them in terms of the *emergence* of individual and collective meanings in the classroom.⁵

A further contrast between the two perspectives concerns the treatment of semiotic mediation. It is important to clarify that the emergent approach fully accepts Vygotsky's (1987) fundamental insight that semiotic mediation is crucially involved in students' conceptual development. The issue under consideration is that of explaining the nature of this involvement. In line with its central tenets, sociocultural accounts of semiotic mediation give precedence to social and cultural

processes over individual psychological processes. For example, in one line of explanation most directly associated with Vygotsky, cultural tools such as conventional mathematical symbols are said to be *internalized* and to become cultural tools for thinking (Davydov & Radzikhovskii, 1985; Rogoff, 1990). In a second line of explanation associated with Leont'ev (1978), individuals are said to *appropriate* cultural tools to their own activity. Both formulations distinguish between students' personal meanings and sociohistorically developed cultural meanings inherent in the appropriate use of cultural tools. Furthermore, both approaches contend that students develop particular culturally approved meanings as they learn to use language and other semiotic means appropriately (cf. Forman, 1996). These approaches, therefore, characterize symbols as primary vehicles of the enculturation because they serve as carriers of meaning from one generation to the next when students use them while engaging in culturally organized activities (van Oers, 1996). It was in this sense that Vygotsky referred to symbols as objective tools (Bauersfeld, 1995). The underlying metaphor is again that of transfer or transmission because learning is characterized as a process in which students inherit the cultural meanings that constitute their intellectual bequest from prior generations.

In the alternative emergent perspective, learning is viewed as a process of both active individual construction and enculturation. Furthermore, processes of signification are considered to be integral to both classroom mathematical practices and the activities of students who participate in them. For example, the mathematical practices established by a classroom community might involve reasoning with physical materials, pictures, diagrams, computer graphics, or notations. An analysis of classroom mathematical practices can, in fact, delineate chains of signification (Walkerdine, 1988) that emerge as classroom mathematical practices evolve (Cobb et al., in press). When attention shifts from collective to individual activity, the physical materials, symbols, and notations that students use are viewed as constituent aspects of their activity rather than as external tools (Bateson, 1973; Dewey, 1977; Prawat, 1995). As a consequence, the use of particular materials and symbols is considered to profoundly influence both the nature of the mathematical capabilities that students develop and the processes by which they develop them.

We contend that the account of signification offered by the emergent approach is better suited to the purposes of developmental research because it provides greater precision than do sociocultural approaches. For example, a sociocultural analysis of a classroom teaching experiment might account for students' learning in terms of their appropriation or internalization of particular semiotic means. The difficulty from our point of view is that such an analysis does not specify in any detail the evolving social situation of the students' development by analyzing instructional sequences as they are realized in interaction in a particular classroom. In addition, this approach tends to downplay qualitative differences in individual children's mathematical interpretations except to

⁵It should be clear from this account that emergent and sociocultural theorists both attribute a proactive role to the teacher in supporting students' mathematical development. We stress this point because the notion that individual students' activity and the norms and practices of the classroom community are reflexively related is sometimes interpreted to mean that students should be given the freedom to construct their own mathematics with minimal, if any, assistance. Such interpretations make a basic category error in our view: A claim about how the relation between individual and collective processes might be characterized in *any classroom* is misread as an instructional prescription. In our view, a teacher who does not attempt to guide the emergence of individual and collective meanings along potentially revisable trajectories that culminate with participation in the mathematical practices of the wider community is abrogating his or her responsibility to the students, the school, and the wider society. We refer the interested reader to Simon (1995) for an account of the teacher's proactive role that is compatible with the metaphor of emergence rather than transmission.

the extent that they can be tied to the students' participation in different out-of-school communities of practice (Confrey, 1995; Hanks, 1991). In contrast, we illustrated when discussing the interpretive framework at the classroom level that an emergent approach addresses both of these issues. Analyses developed from this perspective, therefore, have implications for both the revision of instructional sequences and the development of follow-up teaching experiments (Cobb et al., in press).

In this discussion, we questioned the relatively common view that a sociocultural stance must be adopted if the central role of language and other semiotic means are to be addressed. As an alternative, we suggested that an emergent approach is appropriate for some purposes because it admits a psychological constructivist view of learning but sees it as inextricably tied to processes of signification (cf. Kaput, 1991; Pirie & Kieren, 1994; Sfard, 1991; Thompson, 1992). An emergent analysis might, in fact, be said to recast appropriation processes posited by sociocultural theorists by focusing on the activities of members of specific classroom communities. What, at the global level of the reproduction of culture, is viewed as a process of transmission becomes, at the local level of the classroom community, a process of emergence in which students' constructive activities and the practices in which they participate are considered to be reflexively related.

Thus far, we focused on situations in which an emergent approach might be particularly relevant. We turn now to consider situations in which a sociocultural perspective is more appropriate, and we do so by first discussing an analysis reported by Crawford (1996). In proposing to view "conscious behavior as a reflection of the socio-cultural environment in which an individual functions" (p. 132), Crawford made it clear that she was taking a strong sociocultural perspective. One of her primary interests was to understand situations in which "there are conflicts and inconsistencies between the values and priorities of cultural experience at home and at school" (p. 134). As an illustration, she discussed the conflicts that arise when children growing up in traditional Aboriginal communities in Australia participate in school mathematics activities.

The resistance of many Aboriginal students to learning mathematics in schools has been interpreted as lack of ability by many educators. In fact, for many Aboriginal people, the value conflicts that arise as a result of the world view that is implicit in the elementary mathematics curriculum are substantial barriers to learning. ... [For example,] the very high priority given in Western culture to quantity and to quantifiable variables was not supported by everyday activities and modes of categorical thinking in traditional Aboriginal communities. (p. 135)

Crawford (1996) went on to observe that "Aboriginal communities find the educational practice, used frequently by teachers of mathematics, of asking students questions when

the answer is already known to the teacher, extremely puzzling and distasteful" (p. 135).

In addition, there are "substantial differences between Aboriginal and non-Aboriginal categorical thinking even about such perceptually grounded concepts as color." As a consequence, for Aboriginal children, "the primary colors were not immediately evident as a means of classification [of manipulative materials]" (p. 135).

We find Crawford's (1996) analysis compelling and suggest that, for her purposes, it would be counterproductive to recast the process by which Aboriginal children appropriate the values and priorities of their communities in the emergent terms. In the analysis, these children are portrayed as "carriers" of the culturally based understandings of their communities. The vantage point that Crawford seems to adopt is, therefore, that of an observer located outside the cultural group. From this perspective, thought and activity within a cultural group appear to be relatively homogeneous when compared with differences between groups. This was also the perspective that we took when conducting the school-level and societal-level analyses. In the case of the teachers at the two action research sites, for example, we viewed them as representatives of different pedagogical communities whose activity reflected the priorities and values of those communities. Similarly, in the comparison of the arithmetical learning in Taiwan and the United States, the children, teachers, and parents in the two countries were viewed as carriers of distinct systems of cultural beliefs and values. In the course of the analysis, we did, in fact, point out the qualitative differences in the mathematical activity of children within each of the two national groups (Yang & Cobb, 1995). However, these observations were tangential to the major emphasis of the analysis and merely served to illustrate the possibility of focusing on the constructive activities of individual children.

Crawford (1996) clarified that situations involving tensions in individuals' needs, expectations, and goals are not limited to conflicts between home and school experience, they also include attempts to reform instruction. In such cases, the tension is between the needs, experiences, and goals of the innovators and the teachers, or between those of the teachers and the students. For example, in the school-level analysis, our interactions with the teachers at the inner-city site can be characterized in terms of a tension between our own and the teachers' culturally situated beliefs about what it means to be a child in school. Further, our experiences of working with the teachers at both action research sites can be seen to involve a tension between our own and the teachers' views about the general nature of mathematical activity in school. In this regard, Crawford observed that teachers tend to teach in the ways in which they were taught. She accounts for this phenomenon in sociocultural terms by contending that future teachers appropriate attitudes and beliefs about how mathematics is learned and about the role of the teacher from their own participation as students in the culturally organized activities of schooling. In conducting an

emergent analysis, we, for our part, would recast this appropriation process. It can be noted, for example, that the beliefs and attitudes to which Crawford referred are the psychological correlates of classroom social and sociomathematical norms. Consequently, from an emergent perspective, future teachers are seen to actively construct the beliefs, suppositions, and assumptions that subsequently find expression in their pedagogical activity when, as students, they participated in the negotiation of classroom social and sociomathematical norms. In this account, a global process of appropriation from the sociocultural environment is recast as one of negotiation and individual construction at the classroom level. More generally, this act of recasting appropriation processes as processes of emergence is, for us, the key to coordinating sociocultural and emergent perspectives. The issue is not which of these two accounts gets things right. Instead, it is to consider the situations in which one type of analysis or the other might be more helpful. In our view, the precision of the emergent account is appropriate for certain purposes. However, in other situations, the global nature of sociocultural accounts has its own advantages. In this respect, the two theoretical perspectives can be seen to complement each other. The sociocultural approach that Crawford illustrated focuses on the social and cultural bases of personal experience, whereas analyses developed from the emergent perspective account for the constitution of social and cultural processes by actively cognizing individuals.

CONCLUSION

We used Crawford's (1996) work as a paradigm case to illustrate the relevance of sociocultural approaches to issues of cultural diversity and reform at a more global level. It should be clear from the discussion that we consider both sociocultural and emergent perspectives to be viable positions. We also note that a central notion common to both perspectives and to psychological constructivism is that of activity. Differences between the perspectives concern the positioning of the researcher and, thus, the way in which activity is framed.

In psychological constructivist approaches, the analytical position taken by the analyst is inside an ongoing interaction, and the focus is on the ways in which individual students reorganize their activity while interacting with others (see Figure 3). The emergent approach coordinates analyses of this type with those conducted from the interactionist perspective. We suggested that the analytical position taken in this latter perspective is that of an observer of ongoing interactions located outside the local community but inside the broader cultural community (see Figure 3). From this vantage point, individual activity is seen to be situated within the practices of a local community such as that constituted by the teacher and students in the classroom. In contrast, the positioning of the sociocultural theorist is outside the cultural group (see Figure 3). From this perspective, individual activity is situated

in broad sociocultural practices, and learning is characterized as a process of internalization or appropriation while participating in these practices.

In the course of the discussion, we clarified that the emergent approach coordinates the psychological constructivist and interactionist perspectives. This led us to suggest that analyses whose primary purpose is psychological should be conducted against the background of an interactionist analysis of the social situation in which the student is acting. The contrasts we drew between the emergent and sociocultural perspectives paid particular attention to the kinds of issues that analyses conducted from each perspective might reasonably address. In addition, we considered how the two perspectives might complement each other. These possibilities are worth pursuing in our view, given that the perspectives together offer characterizations of individual student's activity, the practices of the classroom community, and those of broader communities of practice. The interpretive framework we outlined represents one attempt to achieve such a coordination.

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