Sequences in GeoGebra

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Summary

In this article we will see the possibilities of the command Sequence of GeoGebra. We will study numerical sequences and sequences of objects in a linear and matrix form. We will study iterated sequences, paying special attention to Fibonacci sequence and fractals.

Introduction

Last summer 2010, it was held a GeoGebra Day² in Salamanca, on September the 9th. We received an advanced course³, which showed us the command Sequence in GeoGebra (version 3.2.44.0). It was the first time I had ever seen this command. I was impressed with the possibilities for this command, and I tried to find applications immediately. One application that I thought about was working with dynamic colours, and another one was working with fractals and iterated sequences.

When I was working with dynamic colours in Sequence I did not get what I expected. I asked my mates about fractals, and they told me that they were not able to construct fractals with Sequence automatically. Other colleagues told me that they were trying to construct the Fibonacci sequence, but they did not succeeded in obtaining it. I accepted the challenge of getting the automatic construction of the Fibonacci sequence, and I got success using the spread sheet. I sent my construction to my colleagues and I received the first automatic fractal (using the spread sheet) in a short time.

After that September, I continued working with the command Sequence and I have generalized some of my constructions. Nowadays, I am a member of a GeoGebra seminar where doubts about the function of the command Sequence were always cropping up. Because of that I decided to the study command. My purpose in this article is to illustrate the possibilities that I have found for this command. The structure of this article is:

1. Numerical sequences

- 2. Sequences of objects
- 3. Friezes
- 4. Matrices and mosaics
- 5. Iterated sequences and fractals
- 6. Limitations and other uses

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² c.f. <u>http://diageogebra.info/</u>

³ <u>http://geometriadinamica.es/gg_day/curso_avanzado.htm</u>

1. Numerical sequence:

In this section we will show the Sequence using it with the numerical sequences. One of the most powerful commands in GeoGebra is the command Sequence. It is a command that generates a list and has the general shape:

Sequence [<Expression>, <Variable>, Start value> , <End Value>,<Increment>]. If it is not specified Increment, it is 1.

The first intuitive use of this command is the creation of numerical sequences, both arithmetic and geometric:

Sequence [a + d n, n, 0, 9], generates {a, a + d, a + 2 d, a + 3 d, ..., a + 9 d}

Sequence [b + r ^ n, n, 0, 9], generates {b, b r, b r ², b r ³, ..., b r⁹}

where a, b, d and r are constants (which could be sliders).

But surely the next step we will want to do is to represent these visually, and we will represent them as points:

Sequence [(n, a + d n), n, 0, 9], generates {(0, a), (1, a + d), (2, a +2 d), (3, a +3 d), ..., (9, a +9 d)}

Sequence [(b r ^ n), n, 0, 9], generates {(0, b), (1, b r), (2, b r ²), (3, b r³), ..., (9, b r⁹)}

Note: If we want to see the function by passing these points, we will just write in the command line: a + dx or br^x



Example: arithmetic sequence

Example: geometric sequence

2. Sequences of objects

In this section we will show sequences of other objects that are not numbers.

Using the command Sequence with points we realize that it does not work only with numbers, but with objects. Some examples may be sequences of polygons, circles, functions, movements...

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Sequence [Polygon[A,B,n], n, 0, 9]
Sequence [Circle [A, n], n, 0, 9]
Sequence [sin(n x), n, 0, 9]
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Sequence [Rotate[object1,A, n 15°], n, 0, 9]

Every command with a numerical parameter can be used inside the command Sequence.

We can use the command Sequence if we want to generalize our constructions.

Other examples of sequences of objects are:



Note: In this example, note that commands quotient and remainder are made with the Bezout's algorithm, and these commands give us results we do not expect when we work with negative values, so we had to make some minor adjustments in these commands.

3. Friezes

In this section we will see the construction of friezes and some problems with the properties of the generated lists.

Some of the examples above get us into the definition of frieze. An example in detail:



Example: frieze

Sequence [Traslate [poly1,n u, n, 0, 5]

Sequence [Traslate[c,n u, n, 0, 5]

Where poly1 is a square of side 2 units, c is a circle and its centre is the centre of poly1, and u is the vector (2,0).

If we try to change properties, such as colour or filling, it changes the properties of all the elements of the list. So if we want to form a frieze with various colours, we create a list for each colour of objects we want to represent.

4. Matrices and mosaics

4.1. Matrices

Here we will see the possibility of working with the command Sequence in matrices.

Another way to work in sequences as if it were a matrix, but without being so is to write all the rows one after another like a single.

For example, to create a matrix of random numbers, we can write in the command line: Sequence [Sequence [RandomBetween [1.6], n, 1, 4], m, 1, 5]

It generates the following matrix: {{1, 4, 5, 2}, {2, 3, 5, 4}, ..., {6, 1, 6, 3}} which is a list with 5 lists of 4 numbers.

But if what we really want is a list of 20 numbers that appear in this list of lists, we could write:

Sequence [RandomBetween [1.6], n, 1, 20]

Sometimes we are sure of what we want to have on the list, and we can use the spreadsheet that includes GeoGebra.

For example, to create a list of random numbers, we can write in the spreadsheet, in cell A1: RandomBetween [1, 6]. Having indicated the cell, place the mouse over the lower left corner and drag down, which will copy the formula we have written in other marked cells.

Pointing to a row or column and using the right mouse button, we can have these values as a list.

If we pointed out a region that can be converted into a matrix, pressing the right mouse button, the program allows us to create a matrix of cells values directly.

To work with the values of these generated lists, use their own commands: Element, First...



Example: random balls



Example: Christmas's tree

4.2. Mosaics

Here we will see how we can construct mosaics using Secuence.

To work with mosaics we use matrices, so the sequences will be nested: Sequence [Sequence [polygon1 + (n, m), n, 1, 4], m, 1, 5]



Example: coloured mosaics



Example: packing of circles in the plane

5. Iterated sequences and fractals

In this section we will discuss the various possibilities of creating an iterated sequence, focusing on the Fibonacci sequence. In addition to this we will see an example of fractal.

With the help of the spreadsheet we can make iterated sequences.



Example: Fibonacci

Now we will see the construction step by step.

Define A1 and A2 cells as 1, define A3 cell as A1+A2 and drag down the A3 cell. Now we have the first values of the Fibonacci sequence in the A column of the spreadsheet. To work with this column like a list, you have to define it.

The first way is just to write in the command line: $\{A1:A10\}$ matrix1 = $\{\{1,1,2,3,5,8,...\}\}$ You have to watch that the list generated automatically has got a double brace. So if we want to represent, for example, the first 10 values as points, we can write: Sequence [(n, Element [Element [matrix1, 1], n]), n, 1, 10] Another way would be to eliminate the double brace: list1 = Element [matrix1, 1] so we have: list1 = {1,1,2,3,5,8,...}

The second way to define the list: select the cells with the values, click the right button and create a list of the Fibonacci sequence: $L_1 = \{1, 1, 2, 3, 5, 8, ...\}$

And finally we represent the values like points: Sequence [(n, Element [list2, n]), n, 1, 10] or Sequence [(n, Element [L₁, n]), n, 1, 10]

Note: In all the examples we have seen, instead of fixing the end of the sequence, we can use a slider.

The Fibonacci's example, introduces us to the fractal concept.



Example: fractal

6. Limitations and other uses

In this section we will see not only the limitations of the command Sequence but also a limitation of GeoGebra that can be solved with the command Sequence.

Limiting the color

As discussed in section 3, if we try to change properties, such as colour or filling, it changes the properties of all the elements of the list.

This makes impossible my first idea about the use of Sequence: to generate a list of points that takes up all the screen view.

Limitation of fractals

If we try that fractals get generated automatically like in Fibonacci's example, we realize the limitations of the program: it takes a long time to draw the graphics. In this case it is better to create a few iterations and a slide show the graphics.

Limitation of redefining

Another limitation I have found in GeoGebra is the redefinition of some sequences. When a slider is a parameter of the command Sequence, and we change its value, sometimes the screen view do not show us what we expect. The screen view need a refresh.



This mistake can be solved pushing Ctrl +Z

To solve movement problems

A line can be defined in two ways: as a geometric object or a function. Example:

· Line through the points A = (0,1) and B = (2,3).

It is generated line a: x-y = -1

 \cdot Write in the command line: x +1 .

It is generated the function f(x) = x + 1

If we try to make a movement with a geometric object we will have no problem. But if we try to make a movement with a function, GeoGebra does not allow it.

To solve this problem with non-geometrical functions, we will do the following

steps:

- 1. To create a sequence of points of the function.
- 2. To create a sequence of segments defined by the above points.
- 3. The polygonal obtained is an approximation to our function.
- 4. This polygonal can be moved because it is defined geometrically.

Note: If we move some grafics of functions, we will get more than one image for each x value. Then it is not a really limitation of GeoGebra, because a function must have only one image for each x value.



Conclusion

As a conclusion, Sequence is one of the most useful and versatile command of GeoGebra, and we can use it if we want to generalize our constructions.

Besides, we can make beautiful friezes and mosaics with geometric motifs.

Finally, we have seen that we can construct iterated sequences and even fractals.

However, we must understand its limitations when we use this command.

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You can find all the examples and pictures in this link: <u>http://www.geogebra.org/en/upload/index.php?&direction=0&order=&directory=fmaizjim</u> enez/english

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