Geometric Modeling as Spatial Thinking Approach among Prospective Teachers

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Abstract: *The use of Dynamic Geometry System in mathematics classes has increased steadily over the past years, connecting several topics in mathematics and applied sciences. This study developed a pilot to promote geometric modeling approaches among prospective teachers in STEM (Science, Technology, Engineering and Mathematics) subjects. Inspired by objects from students' lives and knowledge, participants built physical prototypes and modeled them using GeoGebra3D software. Modeling these machines digitally contributed to students' understanding of concepts in geometry and highlight previous held misconceptions. In addition, these activities helped to improve students' spatial thinking and assisted their transition from plane to space while using the software tools. All students' models are available freely on the GeoGebra Materials online platform and particular samples and approaches will be outlined in this work.*

1. Introduction

There are some spatial geometry relationships are not necessarily intuitive for students and teachers. Examples for such non-intuitive relationships could be the visualization of specific relationships between geometric objects and defining particular dependency among them. To tackle some of these difficulties, for instance, the geometry curriculum document of NTCM[1] suggests that a variety of tools should be used for different representations of two- and three-dimensional geometric objects. However, research shows that often teachers still not feel comfortable using technology in their classes and DGS to explore geometric objects with movements (see [5] and [7]). To better understand such kinesthetic representations and their cognitive difficulties the three types of cognitive process proposed by Duval (1998) are followed: *visualization*, *construction* and *reasoning*. This paper reports on and analyses a work of prospective teachers using DGS and the corresponding physical representations of objects with movements.

In our study, prospective teachers were asked to reconstruct digital representations of seesaws based on the physical models developed by them earlier. Initially, the project aimed to assist the participants to investigate joint connections and its circular movements. This activity highlighted the necessity for understanding spatial geometric ideas for developing their suitable digital representations. The

[1] NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
http://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Geometry/

visualization of physical objects digitally and their constructions challenged participants to carefully think about preserving coherence between physical and digital representations and through this engage in clear reasoning with their peers. While physical models were constructed with diverse materials the digital ones were modeled using either Blender or GeoGebra software. In this paper, the focus is on the interaction between physical and digital modeling and especially on how the software environment can support users in strengthening (or forming) their mathematical concepts. In this context, this work employed theoretical approaches related to semiotic mediation and geometric modeling resources for multiple representations.

Following the research question *"How can prospective teachers use the combination of physical and digital geometric modeling of joints to promote interdisciplinary and multiple solution strategies for mathematics learning?"* we consider the "how" in order to broaden the lens on the construction of spatial geometric objects. Then we focused the main question into a more practical approach, aiming to investigate students' visual perceptions and mathematical interpretations from their interactions with either physical constructions or using GeoGebra to develop geometrical constructions.

We hope that this approach can offer insights into how the selection of the tools and the design of DGS-based tasks can support prospective teachers enhancing students' learning and their teaching process. We also aim to show how particular models construction enables the development of geometric argumentations.

2. Theoretical Framework

The contextualized treatment of knowledge connecting STEM areas is becoming important for the teaching and learning of STEM disciplines. In our digital modeling task, for instance, geometric and instrumental knowledge are simultaneously explored and guided through by teachers. During the tasks students need to develop and understand logical conditionals to simulate the objects actions as well as they need to comprehend and represent simple mechanical principles. Furthermore, students must explore and interact with the tools of the software, their peers in the team, and the teacher and this approach supports their confidence in learning. We overviewed several theoretical frameworks that helped us developing such guidelines for our study.

2.1. DGS, Instrumental Approach and Semiotic Mediation

The emergence of Dynamic Geometric Systems (DGSs) complemented and supported first traditional geometry courses, but later they spread to various applications of geometry in science and daily life. Several studies (e.g. Schumann, 2004; Gawlick, 2005; Bu & Hohenwarter, 2015) highlighted how DGS supported changes in mathematics teaching and learning as well as contributed to practices of professional mathematicians. However, these studies also highlighted that it is important to sensibly integrate DGS into education to support and not to damage students' learning.

According to Bussi and Mariotti (2008), Rabardel's instrumental approach is based on the distinction between artifact and instrument, while the artifact can exist by itself, the instrument is a mixed entity with two components: the artifact produced for the subject, and the associated schemes of use, which are the results of a construction of the subject itself or of an appropriation of already existing schemes of use. The instrument describes an artifact and its constituted in the use(s) that the subject develops. In this way, the uses of the artifact also depend on the needs and objectives of the user. According to Stormowski (2015) the importance of transforming a software, in our case GeoGebra, into an instrument is an evolution of the reorganization and modification of user schemes, structuring of the teachers' actions, and relations to mathematical concepts. In our particular case, participants were able to explain the feedback from the software even that the procedure was not exactly how they expected. We observed that GeoGebra contributed to participants' reflections on the posed questions and reorganization of their thinking schemes. As pointed out by Sinclair and Robutti (2012), if participants 'internalize' (Vygotsky, 1978) the use of a DGS the artifact becomes a mean of semiotic mediation (Mariotti, 2010) and that offers opportunities to resolve problems. The Semiotic Mediation Theory is centered around the seminal idea of semiotic mediation introduced by Vygotsky (1978) and it aims to describe and explain the process that starts with the student's use of artifacts and leads to the student's appropriation of a particular mathematical contents (Mariotti & Maracci, 2012).

Finally, English at al. (2008) suggest that model development tends to involve the gradual sorting out, clarifying, revising, refining, and integrating conceptual systems that are at intermediate stages of development. This is regardless whether the problem solver is an individual or a group. However, the mediation has a significant role in identifying limits and potentialities to explore the learning process.

3. The Context

Our experiments were conducted with 19 volunteer prospective mathematic teachers in Brazil. Most of them (14) are in the second and third year of their teacher development program. Two participants are in the first year, while the other three are in the fourth year. All participants were already introduced to GeoGebra during an introductory course of ICT, where they explored the main functionalities and features of the software. Besides this course, they previously enrolled into geometry courses (Plane Geometry, Spatial Geometry and Analytic Geometry) in the beginning of their teacher-training program. In these geometric courses whether or not using any software is a professor's option.

Participants were assigned to 10 teams (two individuals, seven doubles and one triple) and they were invited to construct a seesaw as a physical model from the materials and technique they wanted. Then, they had to reproduce the seesaw models using a Dynamical Geometry Software as correctly as possible. During participants work we conducted observations and the shared discussion of their interactions were used to collect data for our study. This part of the teaching process took months to carry out. Some samples of students' work are shown in Figure 1:

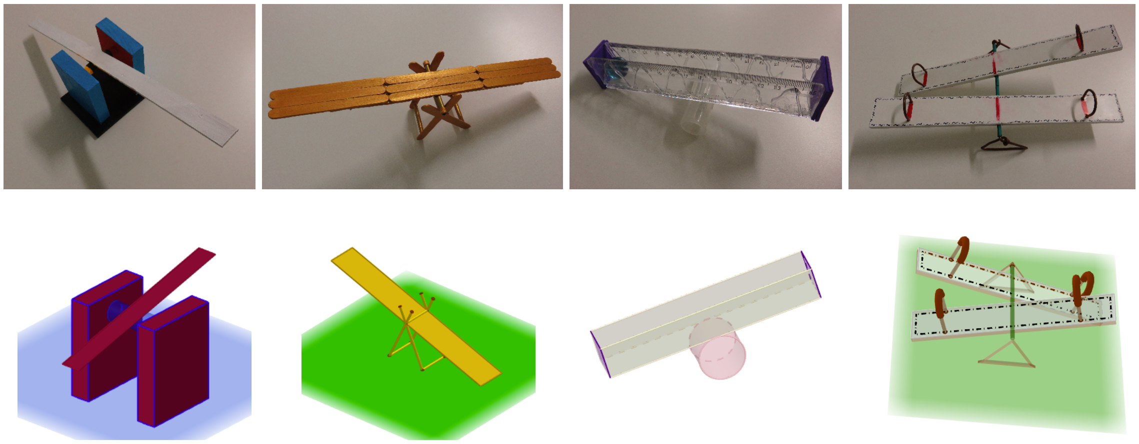


Figure 1: seesaws developed by five different teams
(available in https://www.geogebra.org/m/Tng4JXDk)

Afterwards, participants were invited to continue development with another digital modeling of a new object by their choice. Three teams continued working on designing such new objects: a fan, a house window and a clothesline floor. Teams were observed every two weeks. By the end of the course participants created a variety of digital objects and based on the observations there were common approaches to their work with the initial seesaw construction.

3.1. Research Method

The project focused on enhancing the teaching and learning process of participants with the use of physical and digital objects, then a Design Experimental approach was adopted to carry part of the study. Doerr and Wood (2006) explained that Design Experimental approaches require several cycles of analysis to improve such educational process and interpretation in multiple levels. The collection and analysis of data is prevalent at all stages of the process and feeds back to each new design stage. At all stages of the study researchers should generate and refine principles, properties and products (or processes) that are contributing to the next stages and will later become essential for final designs.

Throughout the project we aimed to encourage discovery based learning and knowledge construction for participants. Ponte (2005) highlighted the importance of such learning approaches and described shifts from the "teaching" activity to the "teaching-learning" activities of such project prticipants. The data collection and analysis reflected on this shift and on the principals of Design Experimental research.

4. Data Analysis and Results

The employed modeling approach in our study covered numerous topics in classical geometry teaching such as similarities, parallel and perpendicular lines, symmetries, rotations, transfers of measurements just to mention a few. These topics are usually taught on paper, but developing physical and digital models, we believe, extended students perspectives on transformation from 2D to 3D. Here we outline two such topics:

4.1. Perpendicular lines

A classical theorem on plane geometry is "*For each point on a line, there exists a unique perpendicular line through that point",* but we cannot transfer the same result to spatial geometry, as there is infinite number of lines passing through such point. In the paper environment students need to imagine it or explained by the teacher. However, in the software environment participants realized it by themselves after repeated unsuccessful attempts to click on perpendicular line icon as well as the symbol and instructions of the 2D and 3D graphics views are different suggesting new learning, see Figure 2:

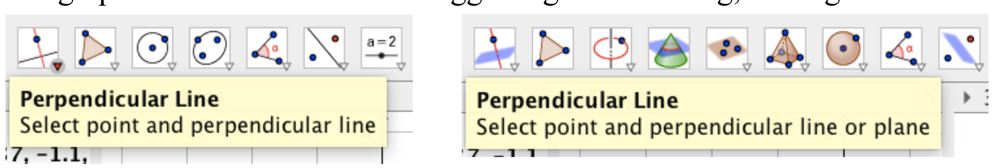
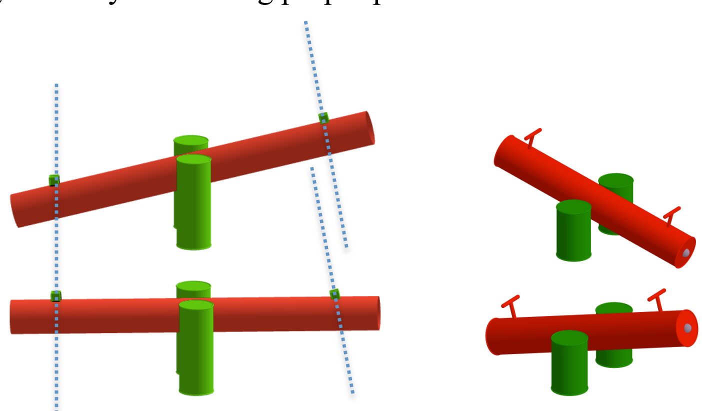


Figure 2: instructions for perpendicular line on 2D view (left) and 3D view (right)

The software itself, though its design, necessitated participants to understand a concept and forced them to find alternative solutions and then construct a plane rather than a line. In addition, the possibility of movements in the software requires students to become more consistent and precise in their geometrical constructions. For instance, during the construction of the rotating board participants could not use static parameters (like parallel to one of the coordinate system axis). Figure 3 (a) shows perpendicular lines that did not fit properly to the board. In Figure 3 (b) the participants managed to construct handles moving correctly and having proper positions.



(a) (b)

Figure 3: seesaw construction in two different moments

Similarly, another team realized during a group presentation that their constructions have incorrect movements and corrected it afterwards.. In the first row of Figure 4, the handle is parallel to the central axis of the seesaw, but in the improved version (second row), they used perpendiculars to the board plane.

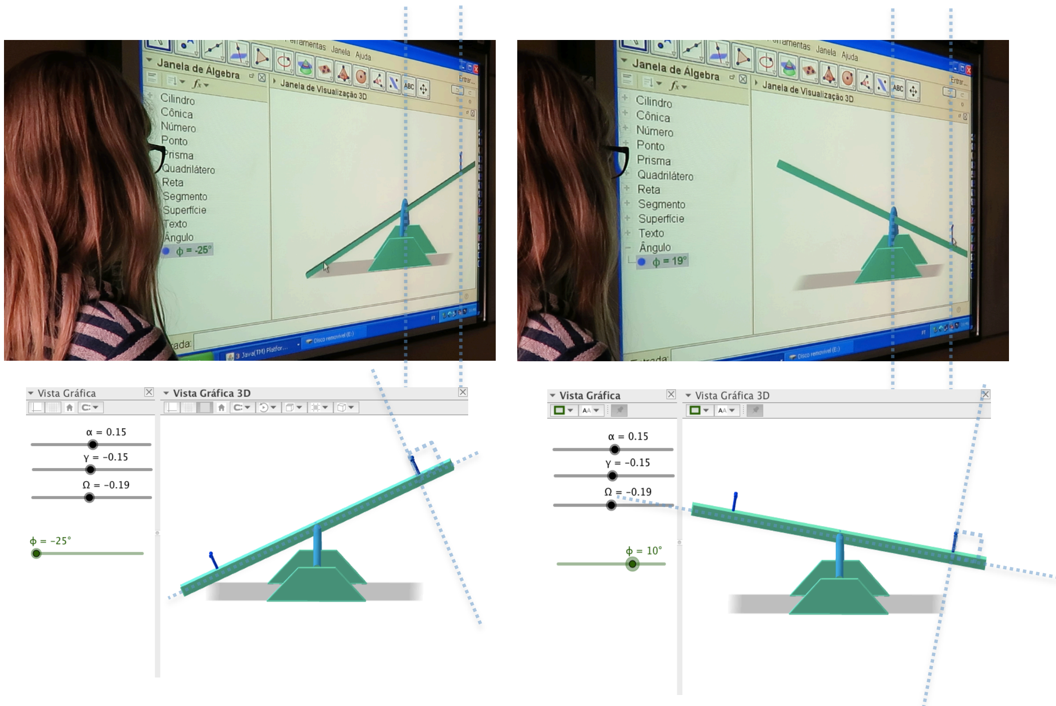


Figure 4: improving the handle position

4.2. Symmetry

To further describe transition from plane to space geometry we observed during the experiment that participants often did not distinguish among the three different types of symmetry (central, line and plane) and how it was supported by their interactions with GeoGebra.

In a stimulated interview two participants were invited to watch their previous construction that was recorded during their work and comment about their ideas and development. The interviews were both screen and video recorded and analyzed offering valuable insight into students' thinking. Our observation of mathematical understanding of students in this special transfer utilized approaches of solving-and-expressing for mathematization processes (Jacinto, Carreira, and Mariotti, 2016; Carreira et al., in press).

Here we report an extract from the recorded interaction with students. First, students used the z-axis to reflect point E, resulting point E'. Then, they reflected E' on the y-axis to obtain point E" (Figure 5). This procedure was successful and, according to them, was inspired by the features, instructions and feedback of the software.

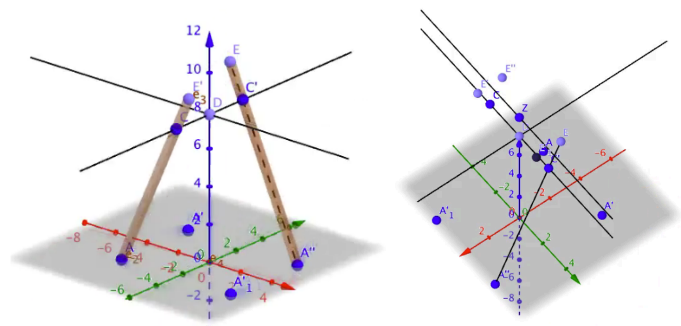


Figure 5: construction with symmetry

1. Researcher: It seems your idea was reflecting the point E around…

2. Student B: …around the Z-axis…

3. Student A: Exactly…

4. Researcher: …and then E' turned up… I would like to know whether this result was according yours expectations or not…when you did this reflection, what exactly you intended to do?

5. Student B: I have already known that reflecting around Z-axis, the point would be created on the opposite quarter, but we actually wanted to reflect either "*x or y direction of symmetry*"…however using the point with one of the two axes as symmetry line, the symmetric point would go down…

6: Student A: it must be with Z, but if we had used plane…

[…]

7. Student B: if we had created the plane x0z or y0Z as we saw at the end we could get the desired effect…

[…]

8. Researcher: …and now you already did this reflection, could you please describe how did the software do that?

[Now the researcher tries to exemplify physically with some pencil and caps] What would be the reflection of this point (pen cap) regarding this line (pencil)?

9. Student A: It would be here (choosing a reasonable position)…

10. Student B: That's right.

11. Researcher: Why?

12. Student B: It seems the idea is to construct a line passing through both elements (the first one and its reflection) and also through the axis we are using as a symmetry line…

13. Student A: that is the middle point (referring to the intersection between the line right defined and the Z-axis).

14. Student A: Actually his intention was to reflect only "regarding x" and eventually GeoGebra did it "*regarding x and then regarding y*", like two simultaneous reflections.

As we can see in the transcript students expectations were different for construction and they could describe it in words consistently. Later on Student B presented an alternative version to explain the movement offered by GeoGebra, but finally admitting that it was not what they intended to do. Although she did not mind the right direction of the pen cap (at this moment it was like a point on space), she described the rotation around the z-axis like a succession of two consecutive planar reflections.

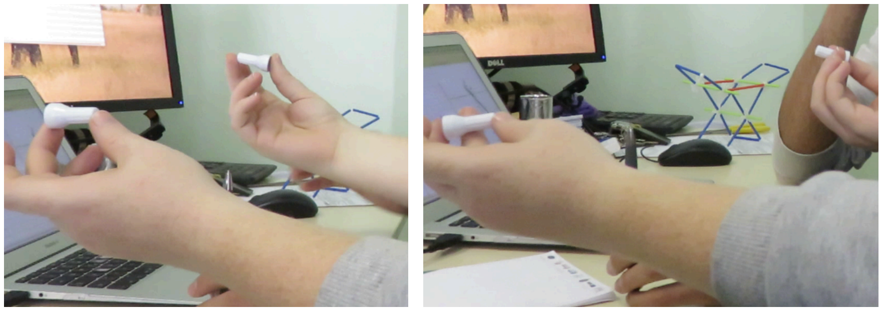


Figure 5: physical representation and explaining the movement that they did digitally

Interesting to note how students used an imprecise language to express themselves. When they use either “around x” (line 14) or “x or y direction of symmetry” (line 5), actually they intended to express the xOz or yOz planes in each case.

Similar events were also registered in the house window construction when participants compared the reflections around a line in space with a reflection around a point on the plane after they followed results given by GeoGebra.

The slider feature of GeoGebra was attractive feature for students to change parameters and to make their constructions interactive both in 2D and 3D environments. It allowed them introducing movements to static objects and offered new ways to their functional thinking. Most of sliders were linked with angle parameters simulating rotational movements. Particular cases used sliders to control a line coefficient and for thickness controller too. The slider tool was used as a calibrator for continuous refinements of constructions. Participants investigated the limitations of construction using the slider first and then define it precisely using synthetic geometry or algebraic support in the mathematization process. Students refined construction criteria with more accurate elements. Also they expose dependencies of analytic geometry in some cases to create symmetric points. Surprisingly, participants used little information from the physical model in the construction of the digital model such as proportions. Most of times they adjusted such proportions by eyes and guessing (trial-and-error).

Finally, a national exam exercise was proposed to reinforce how they interpret (or reinterpret) the seesaw movement with and without technology support. Moreover, how they were able in adapting the exercise over their constructions and analyze geometric behaviors with mechanical movements as well as visualizing 3D objects and spaces from different perspectives. It was good opportunity to promote discussions about loci and start parallel investigations. From the results of these experiments and data analysis we are now creating new methods and assessment for another cohort of pre- and in-service teachers.

5. Discussions

In the previous sections, we described some exercises of developing physical and digital modeling among prospective teachers in Brazil to further develop geometric modeling. The combined use of physical and digital resources representing joints helped us to evaluate perspectives for developing resources and pedagogies for STEM teaching and learning In the examples, we reflected on the experiences of teacher students on developing resources and teaching topics in an unusual environment and with the use of technology.

We observed that student teachers had difficulties expressing and analyzing whether statements like '*There is only one plane parallel to a line passing through a point outside this line*' or *'Two perpendicular lines regarding a third line are both parallels between them'*. It is still more complicated since some statements may assume distinct values (true or false) depending of the context, plane or space, like the second one, for instance. However, our activities with the modeling exercises provided an opportunity for prospective teachers to explore and discover various possibilities with GeoGebra, confirming or refusing their hypothesis. According to the Semiotic Mediation Theory, we could better justify GeoGebra is beyond an artifact and based on students' reflections it could become an instrument. The mathematical inconstancies in some constructions were gradually improved by frequent discussions in groups and occasional interventions from instructors. This is in line with the theory, since the semiotic potential of an artifact consists in the double relationship that occurs between an artifact. but personal meanings emerging from its use to accomplish a task (instrumented activity). Furthermore, mathematical meanings evoked by the use of physical and digital modeling and student showed substantial improvements and expertise in the topics similarly as Bussi and Mariotti (2008) found.

We should also mention that there is not a correct answer for any of these modeling problems, but it rather an exploratory and discovery process enabling students to calibrate their thinking. The outlined examples are just parts of a wider set of exercises and in later papers we will further report on new insights into our physical and digital modeling approaches.

Aknowledgment: This work was supported in part by Federal Institute of Education, Science and Technology of Bento Gonçalves, Brasil, and Johannes Kepler University, in Linz, Austria.

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